# Numerical Results

In this chapter, we present the numerical results derived from both the analytical model and the simulation model, covering four distinct blockchain queuing scenarios. Each scenario has two queues, which are customer queue and the block queue, operating under varying combinations of customer class structures and impatience behavior.

The first scenario models a single-class customer system without impatience, serving as a baseline configuration with First-Come-First-Served (FCFS) discipline and no abandonment. The second scenario introduces two classes of customers with non-preemptive priority, distinguishing high-priority and low-priority customers in both queuing and service procedures. In the third scenario, we revisit the single-class setting while incorporating impatience behavior, where customers may abandon the system after a random impatience threshold. Finally, the fourth scenario combines both priority and impatience, modeling a two-class customer system with distinct abandonment rates and non-preemptive priority rules.

Across all scenarios, the blockchain service is influenced by ON/OFF operational states and employs a partial batch policy during block generation. Performance metrics such as throughput, blocking probability, waiting time, and impatient rate (where applicable) are computed and compared under varying parameter configurations. These results provide a comprehensive view of how priority, impatience, and system reliability jointly impact the overall performance of blockchain-based queuing systems.

## Scenario 1: Single-Class Customer without Impatience

The default values are as provided as below: , , , . The maximum capacity of the system is , and the maximum block size is .

### Block size

Figure 5‑1 to Figure 5‑6 show the relationship between various performance metrics and the block size . Both simulation results and analytical results are shown for comparison.

Figure 5‑1 illustrates the impact of the block size on the average waiting time in the customer queue (). As increases, decreases significantly, particularly at smaller block size. This is because larger blocks allow more customers to be served per service cycle, thereby reducing the time customers spend waiting in the queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑2 illustrates the impact of the block size on the average waiting time in the block queue (). As increases, remains nearly constant. This indicates that the time each block spends in the consensus queue is determined by the consensus rate and system state transition rate, and is independent of block size. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑3 illustrates the impact of the block size on the average waiting time in the system (). As increases, decreases significantly, particularly at smaller block size. This is because larger blocks allow more customers to be served per service cycle, thereby reducing the time customers spend waiting in the queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑4 illustrates the impact of the block size on the average number of customers in the block queue (). As increases, initially grows and then stabilizes around constant value. This indicates that although larger blocks permit more customers per batch, the average block occupancy tends to saturate when the customer arrival rate is equal to the effective service rate of the customer queue, where the effective service rate of the customer queue is dependent on the block generation rate, the system transition rate, and the average block size. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑5 illustrates the impact of the block size on the blocking probability (). As increases, steadily decreases. This is because larger blocks allow more customers to be served per block generation cycle, thereby reducing the chance of the customer queue reaching its capacity limit. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑6 illustrates the impact of the block size on the system throughput (). As increases, rises rapidly at first and then gradually saturates. This is because larger blocks enable more customers to be processed per consensus cycle, but the throughput eventually approaches a limit determined by the customer arrival rate, which is less than the system processing capacity. The system processing capacity is decided by the maximum block size, the block generation rate, consensus rate, and system state transition rate. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑1: Effect of block size on average waiting time in the customer queue

Figure 5‑2: Effect of block size on average waiting time in the block queue

Figure 5‑3: Effect of block size on average waiting time in the system

Figure 5‑4: Effect of block size on average number of customers in block queue

Figure 5‑5: Effect of block size on blocking probability

Figure 5‑6: Effect of block size on system throughput

### Arrival rate

Figure 5‑7 to Figure 5‑12 show the relationship between various performance metrics and the arrival rate . Both simulation results and analytical results are shown for comparison.

Figure 5‑7 illustrates the impact of the arrival rate on the average waiting time in the customer queue (). As increases, increases steadily. This is because higher arrival rate leads to more customers entering the system, causing increased congestion and longer queuing delays before customers can be batched into blocks. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑8 illustrates the impact of the arrival rate on the average waiting time in the block queue (). As increases, remains nearly constant. This indicates that the time each block spends in the consensus queue is determined by the consensus rate and system transition rate, and is independent of the arrival rate. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑9 illustrates the impact of the arrival rate on the average waiting time in the system (). As increases, increases steadily. This is because higher arrival rate leads to more customers entering the system, causing increased congestion and longer queuing delays before customers depart from the system. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑10 illustrates the impact of the arrival rate on the average number of customers in the block queue (). As increases, increases steadily. This is because a higher arrival rate leads to more frequent block formation and more customers being accumulated in each block, increasing the average number of customers waiting for consensus. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑11 illustrates the impact of the arrival rate on the blocking probability (). As increases, rises sharply, specially beyond . This is because a higher arrival rate leads to more customers being accumulated in the customer queue, increasing the chance that incoming customers are blocked due to limited queue capacity. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑12 illustrates the impact of the arrival rate on the system throughput (). As increases, rises rapidly at first and then gradually saturates. This is because higher arrival rates supply more customers into the system, but throughput eventually becomes limited by the system service capacity, which is decided by the maximum block size, the block generation rate, the consensus rate, and system transition rate. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑7: Effect of arrival rate on average waiting time in the customer queue

Figure 5‑8: Effect of arrival rate on average waiting time in the block queue

Figure 5‑9: Effect of arrival rate on average waiting time in the system

Figure 5‑10: Effect of arrival rate on average number of customers in block queue

Figure 5‑11: Effect of arrival rate on blocking probability

Figure 5‑12: Effect of arrival rate on throughput

### Block generation rate

Figure 5‑13 to Figure 5‑18 show the relationship between various performance metrics and the block generation rate . Both simulation results and analytical results are shown for comparison.

Figure 5‑13 illustrates the impact of the block generation rate on the average waiting time in the customer queue (). As increases, decreases significantly. This is because higher block generation rates allow customers to be grouped and processed more frequently, which reduces congestion in the customer queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑14 illustrates the impact of the block generation rate on the average waiting time in the block queue (). As increases, remains nearly constant. This indicates that the time each block spends in the consensus queue is determined by the consensus rate and system transition rate, and is independent of the block generation rate. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑15 illustrates the impact of the block generation rate on the average waiting time in the system (). As increases, decreases steadily. This is because higher block generation rates allow customers to be grouped and processed more frequently, which reduces congestion in the customer queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑16 illustrates the impact of the block generation rate on the average number of customers in the block queue (). As increases, initially grows and then stabilizes around constant value. This indicates that although more frequent block generation can expedite customers service, the average block occupancy tends to saturate because of the arrival rate and the effective service rate reached a state of equilibrium. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑17 illustrates the impact of the block generation rate on the blocking probability (). As increases, decreases rapidly. This is because higher block generation rates allow customers to be served more frequently, reducing the possibility that the customer queue reaches its capacity and causes incoming arrivals to be blocked. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑18 illustrates the impact of the block generation rate on the system throughput (). As increases, rises rapidly at first and then gradually saturates. This is because higher block generation rates enable more customers to be processed per consensus cycle, but the throughput eventually approaches a limit determined by the customer arrival rate, which is less than the system processing capacity. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑13: Effect of block generation rate on average waiting time in the customer queue

Figure 5‑14: Effect of block generation rate on average waiting time in the block queue

Figure 5‑15: Effect of block generation rate on average waiting time in the system

Figure 5‑16: Effect of block generation rate on average number of customers in block queue

Figure 5‑17: Effect of block generation rate on blocking probability

Figure 5‑18: Effect of block generation rate on system throughput

### Consensus rate

Figure 5‑19 to Figure 5‑24 show the relationship between various performance metrics and the consensus rate . Both simulation results and analytical results are shown for comparison.

Figure 5‑19 illustrates the impact of the consensus rate on the average waiting time in the customer queue (). As increases, decrease steadily. This is because a higher consensus rate shortens the time block spend in the consensus queue, enabling faster turnover and more frequent admission of waiting customers from the customer queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑20 illustrates the impact of the consensus rate on the average waiting time in the block queue (). As increases, decrease significantly. This is because a faster consensus rate allows blocks to be processed more quickly, thereby reducing the time that blocks spend waiting in the queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑21 illustrates the impact of the consensus rate on the average waiting time in the system (). As increases, decrease significantly. This is because faster consensus processing reduces delays in the block queue, which in turn accelerates the overall service flow and shortens the total time customers spend in the system. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑22 illustrates the impact of the consensus rate on the average number of customers in the block queue (). As increases, decrease significantly. This is because a higher consensus rate enables faster processing of blocks, which reduces queue accumulation and lowers the . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑23 illustrates the impact of the consensus rate on the blocking probability (). As increases, decrease rapidly. This is because faster consensus rate enables faster processing of blocks, and lowers the probability of reaching the queue capacity that triggers blocking. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑24 illustrates the impact of the consensus rate on the system throughput (). As increases, increase rapidly at first and then gradually saturates. This is because higher consensus rate allows blocks to be processed more efficiently, but the throughput eventually approaches a limit determined by the customer arrival rate, which is less than the system processing capacity. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑19: Effect of consensus rate on average waiting time in the customer queue

Figure 5‑20: Effect of consensus rate on average waiting time in the block queue

Figure 5‑21: Effect of consensus rate on average waiting time in the system

Figure 5‑22: Effect of consensus rate on average number of customers in block queue

Figure 5‑23: Effect of consensus rate on blocking probability

Figure 5‑24: Effect of consensus rate on system throughput

### Transition rate (from ON to OFF)

Figure 5‑25 to Figure 5‑30 show the relationship between various performance metrics and the transition rate from ON to OFF . Both simulation results and analytical results are shown for comparison.

Figure 5‑25 illustrates the impact of the transition rate on the average waiting time in the customer queue (). As increases, increases steadily. This is because more frequent transitions from ON to OFF reduce the availability of block generation service, causing longer queueing delays for arriving customers. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑26 illustrates the impact of the transition rate on the average waiting time in the block queue (). As increases, increase steadily. This is because more frequent service interruptions delay consensus processing, resulting in longer waiting times for customer(s) in block queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑27 illustrates the impact of the transition rate on the average waiting time in the system (). As increases, increase steadily. This is because more frequent service interruptions caused by transitions to the OFF state reduce overall availability of block generation and consensus service, leading to longer queueing delays for customers in the system. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑28 illustrates the impact of the transition rate on the average number of customers in the block queue (). As increases, increases steadily. This is because more frequent service interruptions of block generation processing cause more customers to wait in the customer queue while forming batches from the customer queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑29 illustrates the impact of the transition rate on the blocking probability (). As increases, increases steadily. This is because more frequent service interruptions reduce the system’s capacity to process customers, which increases the probability that the customer queue reaches its capacity and causes incoming arrivals to be blocked. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑30 illustrates the impact of the transition rate on the system throughput (). As increases, decreases gradually. This is because more frequent service interruptions reduce the chance for block generation and consensus processing, thereby limiting the rate at which customers are served and ultimately lowering the overall system throughput. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑25: Effect of transition rate on average waiting time in the customer queue in the system

Figure 5‑26: Effect of transition rate on average waiting time in the block queue

Figure 5‑27: Effect of transition rate on average waiting time in the system

Figure 5‑28: Effect of transition rate on average number of customers in block queue

Figure 5‑29: Effect of transition rate on blocking probability

Figure 5‑30: Effect of transition rate on system throughput

## Scenario 2: Two-Class Customer without Impatience

The default values are as provided as below: , , , , , , . The maximum capacity of the system is , and the maximum block size is .

### Block size

Figure 5‑31 to Figure 5‑36 show the relationship between various performance metrics and the block size . Both simulation results and analytical results are shown for comparison.

Figure 5‑31 illustrates the impact of the block size on the average waiting times in the customer queue for high-priority, low-priority, and overall customers. As increases, the decreases. The reduction is more pronounced for , while remain consistently low. This is because larger blocks allow more customers to be served per service cycle, thereby reducing the time, especially for low-priority customers who tend to experience longer delays when is small. In addition, the is much smaller than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑32 illustrates the impact of the block size on the average waiting times in the block queue for high-priority, low-priority, and overall customers. As increases, the average waiting time in the block queue remains nearly constant for all priority levels. This indicates that the time each block spends in the consensus queue is determined by the consensus rate and system state transition rate, and is independent of block size. In addition, the is smaller than . This is because is larger than . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑33 illustrates the impact of the block size on the average waiting times in the system for high-priority, low-priority, and overall customers. As increases, the decreases. The decline is especially significant for low-priority customers, while the remains relatively constant. This is because larger blocks allow more customers to be served per service cycle, which benefits low-priority customers who are otherwise delayed by the non-preemptive priority mechanism. In addition, the is much smaller than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue and is larger than . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑34 illustrates the impact of the block size on the average numbers of customers in the block queue for high-priority, low- priority, and overall customers. As increases, the average number of customers in the block queue rises gradually across all priority levels. The increase is most noticeable for and , while the remains relatively low and stable. This indicates that although larger blocks permit more customers per batch, the average block occupancy tends to saturate when the customer arrival rate is equal to the effective service rate of the customer queue. In addition, is smaller than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue and the high-priority customers have faster consensus rate than low-priority customers in the block queue, and therefore more low-priority customers remain waiting in the customer queue before being batched. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑35 illustrates the impact of the block size on the blocking probabilities for high-priority, low- priority, and overall customers. As increases, the blocking probability decreases across all priority levels. The decline is more pronounced for , which is initially much higher and drops significantly with increasing . This is because larger blocks allow more customers to be served per block generation cycle, thereby reducing the chance of the customer queue reaching its capacity limit, especially for low-priority customers who are more likely to be blocked under limited queue capacity. In addition, is smaller than . This is because the capacity limit of the customer queue for the high-priority customers is no smaller than that for low-priority customers. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑36 illustrates the impact of the block size on the system throughputs for high-priority, low-priority, and overall customers. As increases, the system throughput increases across all priority levels and then gradually saturates. Both and increase with , with the growth being more significant for low-priority customers. This is because larger blocks enable more customers to be processed per consensus cycle. However, the throughput eventually approaches a limit determined by the customer arrival rate, which is less than the system processing capacity. In addition, is smaller than . This is because the customer arrival rate of the high-priority customers is smaller than that of low-priority customers. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑31: Effect of block size on average waiting time in the customer queue

Figure 5‑32: Effect of block size on average waiting time in the block queue

Figure 5‑33: Effect of block size on average waiting time in the system

Figure 5‑34: Effect of block size on average number of customers in block queue

Figure 5‑35: Effect of block size on blocking probability

Figure 5‑36: Effect of block size on system throughput

### Arrival rate

Figure 5‑37 to Figure 5‑42 show the relationship between various performance metrics and the arrival rate of high-priority customers . Both simulation results and analytical results are shown for comparison.

Figure 5‑37 illustrates the impact of the arrival rate of high-priority customers on the average waiting times in the customer queue for high-priority, low-priority, and overall customers. As increases, the increases steadily. The rise is mainly due to the significant increase in , while remains nearly constant. This is because more high-priority arrivals dominate the queue under the non-preemptive priority mechanism, causing low-priority customers to wait longer in the customer queue. In addition, the is much shorter than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑38 illustrates the impact of the arrival rate of high-priority customers on the average waiting times in the block queue for high-priority, low-priority, and overall customers. As increases, and remains nearly constant. This indicates that the time each block spends in the consensus queue is determined by the associated consensus rate and system transition rate, and is independent of . In addition, is smaller than . This is because is larger than . Furthermore, as increases, decreases. This is because as increases, more high-priority blocks are formed and the consensus rate of high-priority customers is larger than that of low-priority customers. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑39 illustrates the impact of the arrival rate of high-priority customers on the average waiting times in the system for high-priority, low-priority, and overall customers. As increases, the increases steadily. The rise is mainly due to the significant increase in , while remains nearly constant. This is because more dominate the queue under the non-preemptive priority mechanism, causing low-priority customers to spend more time in the system. In addition, is much smaller than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue and is larger than . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑40 illustrates the impact of the arrival rate of high-priority customers on the average numbers of customers in the block queue for high-priority, low-priority, and overall customers. As increases, the remains relatively stable, but with diverging trends across priority class. Specifically, increases and decreases. This behavior reflects the shift in queue composition under the non-preemptive priority mechanism, where more high-priority customers are admitted into the system while low-priority customers are blocked earlier or delayed at the customer queue. Thus, more high-priority blocks and less low-priority blocks are formed. In addition, is smaller than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue and the high-priority customers have faster consensus rate than low-priority customers in the block queue, and therefore more low-priority customers remain waiting in the customer queue before being batched. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑41 illustrates the impact of the arrival rate of high-priority customers on the blocking probabilities for high-priority, low-priority, and overall customers. As increases, the blocking probability increases across all priority levels. The rise is most significant for , who face greater difficulty being admitted into the system due to the increased presence of high-priority arrivals. This trend reflects the effect of the non-preemptive priority mechanism, where high-priority customers dominate the queue and are less probability to be blocked, while low-priority customers experience higher blocking rates as system congestion intensifies. In addition, is smaller than . This is because the capacity limit of the customer queue for the high-priority customers is no smaller than that for low-priority customers. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑42 illustrates the impact of the arrival rate of high-priority customers on the system throughputs for high-priority, low-priority, and overall customers. As increases, increases and decreases. The remains relatively stable, as the gain in compensates for the loss in . This behavior reflects the shift in resource allocation under the non-preemptive priority mechanism, where increasing leads to more system capacity being devoted to high-priority customers at the expense of low-priority ones. In addition, is smaller than . This is because the customer arrival rate of the high-priority customers is smaller than that of low-priority customers. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑37: Effect of arrival rate on average waiting time in the customer queue

Figure 5‑38: Effect of arrival rate on average waiting time in the block queue

Figure 5‑39: Effect of arrival rate on average waiting time in the system

Figure 5‑40: Effect of arrival rate on average number of customers in block queue

Figure 5‑41: Effect of arrival rate on blocking probability

Figure 5‑42: Effect of arrival rate on throughput

### Block generation rate

Figure 5‑43 to Figure 5‑48 show the relationship between various performance metrics and the block generation rate of high priority customers . Both simulation results and analytical results are shown for comparison.

Figure 5‑43 illustrates the impact of the block generation rate of high-priority customers on the average waiting times in the customer queue for high-priority, low-priority, and overall customers. As increases, the average waiting time in the customer queue decreases across all priority levels. The decline is more substantial for low-priority customers, who benefit from the increased service opportunities enabled by faster block generation. Although high-priority customers also experience shorter waiting times, their improvement is less pronounced since their queuing delay is already low. In addition, the is much smaller than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑44 illustrates the impact of the block generation rate of high-priority customers on the average waiting times in the block queue for high-priority, low-priority, and overall customers. As increases, the average waiting time in the block queue remains nearly constant across all priority levels. This indicates that the time each block spends in the consensus queue is determined by the associated consensus rate and system transition rate, and is independent of the block generation rate. In addition, the is smaller than . This is because is larger than . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑45 illustrates the impact of the block generation rate of high-priority customers on the average waiting times in the system for high-priority, low-priority, and overall customers. As increases, average waiting time in the system decreases across all priority levels. The rise is primarily contributed by , while high-priority customers also benefit from more frequent block formation. This is because higher block generation rates allow customers to be grouped and processed more frequently, which reduces congestion in the customer queue. In addition, the is much smaller than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue and is larger than . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑46 illustrates the impact of the block generation rate of high-priority customers on the average numbers of customers in the block queue for high-priority, low-priority, and overall customers. As increases, the gradually increases. The rise is primarily contributed by , while increases only slightly and remains at a relatively low level. This is because a higher allows high-priority customers to be processed more quickly, which indirectly leads to more low-priority customers forming batches. In addition, is smaller than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue and therefore more low-priority customers remain waiting in the customer queue before being batched. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑47 illustrates the impact of the block generation rate of high-priority customers on the blocking probabilities for high-priority, low-priority, and overall customers. As increases, the blocking probability decreases across all priority levels. This is because the higher block generation rate allows high-priority customers to be served more frequently, which in turn release the capacity in the customer queue for low-priority customers. As a result, the probability that low-priority customers are blocked is reduced. In addition, is smaller than . This is because the capacity limit of the customer queue for the high-priority customers is no smaller than that for low-priority customers. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑48 illustrates the impact of the block generation rate of high-priority customers on the system throughputs for high-priority, low-priority, and overall customers. As increases, the system throughput increases across all priority levels and then gradually saturates. The rise is primarily contributed by , which increases more significantly due to the release of queue capacity made possible by faster processing of high-priority blocks. In contrast, eventually approaches a limit determined by . In addition, is smaller than . This is because the customer arrival rate of the high-priority customers is smaller than that of low-priority customers. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑43: Effect of block generation rate on average waiting time in the customer queue

Figure 5‑44: Effect of block generation rate on average waiting time in the block queue

Figure 5‑45: Effect of block generation rate on average waiting time in the system

Figure 5‑46: Effect of block generation rate on average number of customers in block queue

Figure 5‑47: Effect of block generation rate on blocking probability

Figure 5‑48: Effect of block generation rate on system throughput

### Consensus rate

Figure 5‑49 to Figure 5‑54 show the relationship between various performance metrics and the consensus rate of high-priority customers . Both simulation results and analytical results are shown for comparison.

Figure 5‑49 illustrates the impact of the consensus rate of high-priority customers on the average waiting times in the customer queue for high-priority, low-priority, and overall customers. As increases, the decreases. The reduction is more pronounced for , while remains relatively stable. This is because higher allows high-priority blocks to complete consensus more quickly, thereby shortening the duration that high-priority customers occupy system capacity. As a result, more capacity becomes available for low-priority customers, so that reducing . In addition, the is much smaller than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑50 illustrates the impact of the consensus rate of high-priority customers on the average waiting times in the block queue for high-priority, low-priority, and overall customers. As increases, the decreases slightly. The decline is primarily contributed by , while remains nearly constant. This is because a higher allows high-priority blocks to complete consensus more quickly, reducing the amount of time the high-priority customers spend waiting in the block queue. Since the consensus rate of low-priority customers is not affected, their waiting time remains unchanged. In addition, at , is noticeably higher than , but the two curves intersect around , after which becomes lower. This crossover occurs because starts lower than the fixed , and increasing improves high-priority processing speed. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑51 illustrates the impact of the consensus rate of high-priority customers on the average waiting times in the system for high-priority, low-priority, and overall customers. As increases, the average waiting time in the system decreases steadily for all priority levels. This is because faster reduces delays in the block queue, thereby improving system efficiency. While also decreases slightly as system capacity is released from high-priority customers. This trend highlights how improving consensus efficiency for high-priority customers can enhance overall system flow and reduce and . In addition, the is much smaller than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑52 illustrates the impact of the consensus rate of high-priority customers on the average numbers of customers in the block queue for high-priority, low-priority, and overall customers. As increases, the slightly decreases. This change is mainly driven by a noticeable reduction in , while gradually increases due to the admission of more low-priority customers as system capacity is released by faster . This reflects how increasing can reduce queue capacity for high-priority customers while indirectly increasing . In addition, is smaller than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue and is larger than , and therefore more low-priority customers remain waiting in the customer queue before being batched. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑53 illustrates the impact of the consensus rate of high-priority customers on the blocking probabilities for high-priority, low-priority, and overall customers. As increases, the blocking probability decreases across all priority levels. This decline is more significant for high-priority customers, whose blocks are processed more rapidly with higher , resulting in fewer delays and fewer blocked arrivals. Also, faster processing of high-priority customers indirectly frees up capacity in the customer queue, reducing the as well. In addition, is smaller than . This is because the capacity limit of the customer queue for the high-priority customers is no smaller than that for low-priority customers. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑54 illustrates the impact of the consensus rate of high-priority customers on the system throughputs for high-priority, low-priority, and overall customers. As increases, the system throughput increases across all priority levels and then gradually saturates. The rise is primarily contributed by , which increases more significantly due to the release of queue capacity made possible by faster processing of high-priority blocks. In contrast, eventually approaches a limit determined by . In addition, is smaller than . This is because the customer arrival rate of the high-priority customers is smaller than that of low-priority customers. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑49: Effect of consensus rate on average waiting time in the customer queue

Figure 5‑50: Effect of consensus rate on average waiting time in the block queue

Figure 5‑51: Effect of consensus rate on average waiting time in the system

Figure 5‑52: Effect of consensus rate on average number of customers in block queue

Figure 5‑53: Effect of consensus rate on blocking probability

Figure 5‑54: Effect of consensus rate on system throughput

### Transition rate (ON to OFF)

Figure 5‑55 to Figure 5‑60 show the relationship between various performance metrics and the transition rate . Both simulation results and analytical results are shown for comparison.

Figure 5‑55 illustrates the impact of the transition rate on the average waiting times in the customer queue for high-priority, low-priority, and overall customers. As increases, the average waiting time in the customer queue rises steadily across all priority levels. The increase is more pronounced for low-priority customers, who are more affected by interruptions in service. This is because more frequent transitions from ON to OFF reduce the availability of block generation service, causing longer queueing delays for arriving customers, especially for customers with lower priority. In addition, the is much smaller than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑56 illustrates the impact of the transition rate on the average waiting times in the block queue for high-priority, low-priority, and overall customers. As increases, the average waiting time in the block queue increase steadily across all priority levels. This is because more frequent service interruptions delay consensus processing, resulting in longer waiting times for customer(s) in block queue. This effect is observed for both high- and low-priority customers, with low-priority customers experiencing slightly longer delay. In addition, the is smaller than . This is because is larger than . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑57 illustrates the impact of the transition rate on the average waiting times in the system for high-priority, low-priority, and overall customers. As increases, the average waiting time in the system increase steadily across all priority levels. The increase is more significant for , while increases slightly. This is because more frequent service interruptions caused by transitions to the OFF state reduce overall availability of block generation and consensus service, leading to longer queueing delays for customers in the system. In addition, the is much smaller than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue and is larger than . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑58 illustrates the impact of the transition rate on the average numbers of customers in the block queue for high-priority, low-priority, and overall customers. As increases, the average number of customers in the block queue increase steadily across all priority levels. This is because more frequent service interruptions of block generation processing cause more customers to wait in the customer queue while forming batches from the customer queue. In addition, is smaller than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue and is larger than , and therefore more low-priority customers remain waiting in the customer queue before being batched. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑59 illustrates the impact of the transition rate on the blocking probabilities for high-priority, low-priority, and overall customers. As increases, the blocking probability increase steadily across all priority levels. The rise is more pronounced for , while increases at a slower rate. This is because more frequent service interruptions reduce the system’s capacity to process customers, which increases the probability that the customer queue reaches its capacity and causes incoming arrivals to be blocked. In addition, is smaller than . This is because the capacity limit of the customer queue for the high-priority customers is no smaller than that for low-priority customers. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑60 illustrates the impact of the transition rate on the system throughputs for high-priority, low-priority, and overall customers. As increases, the system throughput decreases gradually across all priority levels. This is because more frequent service interruptions reduce the chance for block generation and consensus processing, thereby limiting the rate at which customers are served and ultimately lowering the . The decline is more pronounced for , whose service opportunities are more severely impacted by limited availability, while remains relatively stable. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑55: Effect of transition rate on average waiting time in the customer queue in the system

Figure 5‑56: Effect of transition rate on average waiting time in the block queue

Figure 5‑57: Effect of transition rate on average waiting time in the system

Figure 5‑58: Effect of transition rate on average number of customers in block queue

Figure 5‑59: Effect of transition rate on blocking probability

Figure 5‑60: Effect of transition rate on system throughput

## Scenario 3: Single-Class Customer with Impatience

The default values are as provided as below: , , , , . The maximum capacity of the system is , and the maximum block size is .

### Block size

Figure 5‑61 to Figure 5‑67 show the relationship between various performance metrics and the block size . Both simulation results and analytical results are shown for comparison.

Figure 5‑61 illustrates the impact of the block size on the average waiting time in the customer queue (). As increases, decreases significantly, particularly at smaller block size. This is because larger blocks allow more customers to be served per service cycle, thereby reducing the time customers spend waiting in the queue. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which further suppresses queue buildup and contributes to the decline in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑62 illustrates the impact of the block size on the average waiting time in the block queue (). As increases, remains nearly constant. This indicates that the time each block spends in the consensus queue is determined by the consensus rate and system state transition rate, and is independent of block size. Furthermore, with impatience is the same as that without impatience. This is because once a block is formed, impatience mechanism has no effect on it. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑63 illustrates the impact of the block size on the average waiting time in the system (). As increases, decreases significantly, particularly at smaller block size. This is because larger blocks allow more customers to be served per service cycle, thereby reducing the time customers spend waiting in the queue. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which further suppresses queue buildup and contributes to the decline in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑64 illustrates the impact of the block size on the average number of customers in the block queue (). As increases, initially grows and then stabilizes around constant value. This indicates that although larger blocks permit more customers per batch, the average block occupancy tends to saturate when the customer arrival rate is equal to the effective service rate of the customer queue, where the effective service rate of the customer queue is dependent on the block generation rate, the system transition rate, and the average block size. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which contributes to a reduction in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑65 illustrates the impact of the block size on the blocking probability (). As increases, steadily decreases. This is because larger blocks allow more customers to be served per block generation cycle, thereby reducing the chance of the customer queue reaching its capacity limit. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which contributes to a reduction in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑66 illustrates the impact of the block size on the system throughput (). As increases, rises rapidly at first and then gradually saturates. This is because larger blocks enable more customers to be processed per consensus cycle, but the throughput eventually approaches a limit determined by the customer arrival rate and the impatient rate, which is less than the system processing capacity. The system processing capacity is decided by the maximum block size, the block generation rate, consensus rate, and system state transition rate. As a result, with impatience is smaller than that without impatience. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑67 illustrates the impact of the block size on the impatience probability (). As increases, steadily decreases. This is because larger blocks allow more customers to be served in each service cycle, which reduces the waiting time in the customer queue. As a result, the probability that customers reach their impatience threshold and leave the system becomes lower. As a result, with impatience is larger than without impatience. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑61: Effect of block size on average waiting time in the customer queue

Figure 5‑62: Effect of block size on average waiting time in the block queue

Figure 5‑63: Effect of block size on average waiting time in the system

Figure 5‑64: Effect of block size on average number of customers in block queue

Figure 5‑65: Effect of block size on blocking probability

Figure 5‑66: Effect of block size on system throughput

Figure 5‑67: Effect of block size on the impatient probability

### Arrival rate

Figure 5‑68 to Figure 5‑74 show the relationship between various performance metrics and the arrival rate . Both simulation results and analytical results are shown for comparison.

Figure 5‑68 illustrates the impact of the arrival rate on the average waiting time in the customer queue (). As increases, increases steadily. This is because higher arrival rate leads to more customers entering the system, causing increased congestion and longer queuing delays before customers can be batched into blocks. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which further suppresses queue buildup and contributes to the decline in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑69 illustrates the impact of the arrival rate on the average waiting time in the block queue (). As increases, remains nearly constant. This indicates that the time each block spends in the consensus queue is determined by the consensus rate and system transition rate, and is independent of the arrival rate. Furthermore, with impatience is the same as that without impatience. This is because once a block is formed, impatience mechanism has no effect on it. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑70 illustrates the impact of the arrival rate on the average waiting time in the system (). As increases, increases steadily. This is because higher arrival rate leads to more customers entering the system, causing increased congestion and longer queuing delays before customers depart from the system. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which further suppresses queue buildup and contributes to the decline in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑71 illustrates the impact of the arrival rate on the average number of customers in the block queue (). As increases, increases steadily. This is because a higher arrival rate leads to more frequent block formation and more customers being accumulated in each block, increasing the average number of customers waiting for consensus. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which contributes to a reduction in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑72 illustrates the impact of the arrival rate on the blocking probability (). As increases, rises sharply, specially beyond . This is because a higher arrival rate leads to more customers being accumulated in the customer queue, increasing the chance that incoming customers are blocked due to limited queue capacity. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which contributes to a reduction in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑73 illustrates the impact of the arrival rate on the system throughput (). As increases, rises rapidly at first and then gradually saturates. This is because higher arrival rates supply more customers into the system, but throughput eventually becomes limited by the impatience rate and the system service capacity, which is decided by the maximum block size, the block generation rate, the consensus rate, and system transition rate. As a result, with impatience is smaller than that without impatience. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑74 illustrates the impact of the arrival rate on the impatience probability (). As increases, increases steadily. This is because a higher arrival rate leads to increased queue congestion and longer waiting times in the customer queue. Consequently, more customers are likely to reach their impatience threshold and leave the system before being served. As a result, with impatience is larger than without impatience. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑68: Effect of arrival rate on average waiting time in the customer queue

Figure 5‑69: Effect of arrival rate on average waiting time in the block queue

Figure 5‑70: Effect of arrival rate on average waiting time in the system

Figure 5‑71: Effect of arrival rate on average number of customers in block queue

Figure 5‑72: Effect of arrival rate on blocking probability

Figure 5‑73: Effect of arrival rate on throughput

Figure 5‑74: Effect of arrival rate on the impatient probability

### Block generation rate

Figure 5‑75 to Figure 5‑81 show the relationship between various performance metrics and the block generation rate . Both simulation results and analytical results are shown for comparison.

Figure 5‑75 illustrates the impact of the block generation rate on the average waiting time in the customer queue (). As increases, decreases significantly. This is because higher block generation rates allow customers to be grouped and processed more frequently, which reduces congestion in the customer queue. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which further suppresses queue buildup and contributes to the decline in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑76 illustrates the impact of the block generation rate on the average waiting time in the block queue (). As increases, remains nearly constant. This indicates that the time each block spends in the consensus queue is determined by the consensus rate and system transition rate, and is independent of the block generation rate. Furthermore, with impatience is the same as that without impatience. This is because once a block is formed, impatience mechanism has no effect on it. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑77 illustrates the impact of the block generation rate on the average waiting time in the system (). As increases, decreases steadily. This is because higher block generation rates allow customers to be grouped and processed more frequently, which reduces congestion in the customer queue. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which further suppresses queue buildup and contributes to the decline in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑78 illustrates the impact of the block generation rate on the average number of customers in the block queue (). As increases, initially grows and then stabilizes around constant value. This indicates that although more frequent block generation can expedite customers service, the average block occupancy tends to saturate because of the arrival rate and the effective service rate reached a state of equilibrium. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which contributes to a reduction in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑79 illustrates the impact of the block generation rate on the blocking probability (). As increases, decreases rapidly. This is because higher block generation rates allow customers to be served more frequently, reducing the possibility that the customer queue reaches its capacity and causes incoming arrivals to be blocked. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which contributes to a reduction in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑80 illustrates the impact of the block generation rate on the system throughput (). As increases, rises rapidly at first and then gradually saturates. This is because higher block generation rates enable more customers to be processed per consensus cycle, but the throughput eventually approaches a limit determined by the customer arrival rate and the impatient rate, which is less than the system processing capacity. As a result, with impatience is smaller than that without impatience. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑81 illustrates the impact of the block generation rate on the impatience probability (). As increases, gradually decreases. A higher block generation rate allows customers to be grouped into blocks and served more frequently, thereby shortening their time in the customer queue. This reduces the chance of reaching the impatience threshold before service. As a result, with impatience is larger than without impatience. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑75: Effect of block generation rate on average waiting time in the customer queue

Figure 5‑76: Effect of block generation rate on average waiting time in the block queue

Figure 5‑77: Effect of block generation rate on average waiting time in the system

Figure 5‑78: Effect of block generation rate on average number of customers in block queue

Figure 5‑79: Effect of block generation rate on blocking probability

Figure 5‑80: Effect of block generation rate on system throughput

Figure 5‑81: Effect of block generation on the impatient probability

### Consensus rate

Figure 5‑82 to Figure 5‑88 show the relationship between various performance metrics and the consensus rate . Both simulation results and analytical results are shown for comparison.

Figure 5‑82 illustrates the impact of the consensus rate on the average waiting time in the customer queue (). As increases, decrease steadily. This is because a higher consensus rate shortens the time block spend in the consensus queue, enabling faster turnover and more frequent admission of waiting customers from the customer queue. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which further suppresses queue buildup and contributes to the decline in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑83 illustrates the impact of the consensus rate on the average waiting time in the block queue (). As increases, decrease significantly. This is because a faster consensus rate allows blocks to be processed more quickly, thereby reducing the time that blocks spend waiting in the queue. Furthermore, with impatience is the same as that without impatience. This is because once a block is formed, impatience mechanism has no effect on it. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑84 illustrates the impact of the consensus rate on the average waiting time in the system (). As increases, decrease significantly. This is because faster consensus processing reduces delays in the block queue, which in turn accelerates the overall service flow and shortens the total time customers spend in the system. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which further suppresses queue buildup and contributes to the decline in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑85 illustrates the impact of the consensus rate on the average number of customers in the block queue (). As increases, decrease significantly. This is because a higher consensus rate enables faster processing of blocks, which reduces queue accumulation and lowers the . Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which contributes to a reduction in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑86 illustrates the impact of the consensus rate on the blocking probability (). As increases, decrease rapidly. This is because faster consensus rate enables faster processing of blocks, and lowers the probability of reaching the queue capacity that triggers blocking. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which contributes to a reduction in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑87 illustrates the impact of the consensus rate on the system throughput (). As increases, increase rapidly at first and then gradually saturates. This is because higher consensus rate allows blocks to be processed more efficiently, but the throughput eventually approaches a limit determined by the customer arrival rate and the impatient rate, which is less than the system processing capacity. As a result, with impatience is smaller than that without impatience. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑88 illustrates the impact of the consensus rate on the system impatient probability (). As increases, steadily decreases. A higher consensus rate makes the consensus process more quickly, thereby lowering the probability of customers in customer queue reaching their impatience threshold. As a result, with impatience is larger than without impatience. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑82: Effect of consensus rate on average waiting time in the customer queue

Figure 5‑83: Effect of consensus rate on average waiting time in the block queue

Figure 5‑84: Effect of consensus rate on average waiting time in the system

Figure 5‑85: Effect of consensus rate on average number of customers in block queue

Figure 5‑86: Effect of consensus rate on blocking probability

Figure 5‑87: Effect of consensus rate on system throughput

Figure 5‑88: Effect of consensus rate on the impatient probability

### Transition rate (from ON to OFF)

Figure 5‑89 to Figure 5‑95 show the relationship between various performance metrics and the transition rate from ON to OFF . Both simulation results and analytical results are shown for comparison.

Figure 5‑89 illustrates the impact of the transition rate on the average waiting time in the customer queue (). As increases, increases steadily. This is because more frequent transitions from ON to OFF reduce the availability of block generation service, causing longer queueing delays for arriving customers. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which further suppresses queue buildup and contributes to the decline in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑90 illustrates the impact of the transition rate on the average waiting time in the block queue (). As increases, increase steadily. This is because more frequent service interruptions delay consensus processing, resulting in longer waiting times for customer(s) in block queue. Furthermore, with impatience is the same as that without impatience. This is because once a block is formed, impatience mechanism has no effect on it. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑91 illustrates the impact of the transition rate on the average waiting time in the system (). As increases, increase steadily. This is because more frequent service interruptions caused by transitions to the OFF state reduce overall availability of block generation and consensus service, leading to longer queueing delays for customers in the system. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which further suppresses queue buildup and contributes to the decline in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑92 illustrates the impact of the transition rate on the average number of customers in the block queue (). As increases, increases steadily. This is because more frequent service interruptions of block generation processing cause more customers to wait in the customer queue while forming batches from the customer queue. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which contributes to a reduction in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑93 illustrates the impact of the transition rate on the blocking probability (). As increases, increases steadily. This is because more frequent service interruptions reduce the system’s capacity to process customers, which increases the probability that the customer queue reaches its capacity and causes incoming arrivals to be blocked. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which contributes to a reduction in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑94 illustrates the impact of the transition rate on the system throughput (). As increases, decreases gradually. This is because more frequent service interruptions reduce the chance for block generation and consensus processing. In addition, due to the impatience mechanism, customers are more likely to abandon the queue when waiting becomes longer, which contributes to a decline in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑95 illustrates the impact of the transition rate on the impatience probability (). As increases, increases steadily. This is because more frequent system on the OFF state which limits service availability, causing customers to wait longer in the queue and thus increasing the probability of reaching their impatience threshold. As a result, with impatience is larger than without impatience. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑89: Effect of transition rate on average waiting time in the customer queue in the system

Figure 5‑90: Effect of transition rate on average waiting time in the block queue

Figure 5‑91: Effect of transition rate on average waiting time in the system

Figure 5‑92: Effect of transition rate on average number of customers in block queue

Figure 5‑93: Effect of transition rate on blocking probability

Figure 5‑94: Effect of transition rate on system throughput

Figure 5‑95: Effect of transition rate on the impatient probability

### Impatient rate

Figure 5‑96 to Figure 5‑102 show the relationship between various performance metrics and the impatient rate . Both simulation results and analytical results are shown for comparison.

Figure 5‑96 illustrates the impact of the impatience rate on the average waiting time in the customer queue (). As increases, gradually decreases. This is because higher impatience rate makes customers more likely to abandon the customer queue once they reach their impatience threshold, thereby shortening the overall queue length and reducing the average waiting time in customer queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑97 illustrates the impact of the impatience rate on the average waiting time in the block queue (). As increases, remains nearly constant. This is because the impatience mechanism only affects customers while they are in the customer queue. Once a block is formed, they are no longer subject to abandonment due to impatience. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑98 illustrates the impact of the impatience rate on the average waiting time in the system (). As increases, steadily decreases. This is because higher impatience rate makes customers more likely abandon the customer queue once they reach their impatience threshold, which reduces congestion and lowers the overall time customers spend in the system. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑99 illustrates the impact of the impatience rate on the average number of customers in the block queue (). As increases, steadily decreases. This is because higher impatience rate makes customers more likely to abandon the customer queue once they reach their impatience threshold, reducing the number of customers that eventually form a block. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑100 illustrates the impact of the impatience rate on the blocking probability (). As increases, gradually decreases. This is because higher impatience rate makes customers more likely abandon the customer queue once they reach their impatience threshold, thereby reducing the probability that queue reaches its capacity and triggers blocking. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑101 illustrates the impact of the impatience rate on the system throughput (). As increases, decreases gradually. This is because higher impatience rate makes customers more likely abandon the customer queue once they reach their impatience threshold, which reduces the total number of customers successfully processed by the system. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑102 illustrates the impact of the impatience rate on the impatience probability (). As increases, increases significantly. This is because higher impatience rate makes customers more likely abandon the customer queue once they reach their impatience threshold. As a result, the proportion of customers abandoning the system increases, leading to a higher impatience probability. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑96: Effect of transition rate on average waiting time in the customer queue in the system

Figure 5‑97: Effect of transition rate on average waiting time in the block queue

Figure 5‑98: Effect of transition rate on average waiting time in the system

Figure 5‑99: Effect of transition rate on average number of customers in block queue

Figure 5‑100: Effect of transition rate on blocking probability

Figure 5‑101: Effect of transition rate on system throughput

Figure 5‑102: Effect of transition rate on the impatient probability

## Scenario 4: Two-Class Customer with Impatience

The default values are as provided as below: , , , , , , , , . The maximum capacity of the system is , and the maximum block size is .

### Block size

Figure 5‑103 to Figure 5‑108 show the relationship between various performance metrics and the block size . Both simulation results and analytical results are shown for comparison.

Figure 5‑103 illustrates the impact of the block size on the average waiting times in the customer queue for high-priority, low-priority, and overall customers. As increases, the decreases. The reduction is more pronounced for , while remain consistently low. This is because larger blocks allow more customers to be served per service cycle, thereby reducing the time, especially for low-priority customers who tend to experience longer delays when is small. In addition, the is much smaller than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue. Furthermore, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which further suppresses queue buildup and contributes to the decline in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑104 illustrates the impact of the block size on the average waiting times in the block queue for high-priority, low-priority, and overall customers. As increases, the average waiting time in the block queue remains nearly constant for all priority levels. This indicates that the time each block spends in the consensus queue is determined by the consensus rate and system state transition rate, and is independent of block size. In addition, the is smaller than . This is because is larger than . Furthermore, with impatience is the same as that without impatience. This is because once a block is formed, impatience mechanism has no effect on it. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑105 illustrates the impact of the block size on the average waiting times in the system for high-priority, low-priority, and overall customers. As increases, the decreases. The decline is especially significant for low-priority customers, while the remains relatively constant. This is because larger blocks allow more customers to be served per service cycle, which benefits low-priority customers who are otherwise delayed by the non-preemptive priority mechanism. In addition, the is much smaller than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue and is larger than . Furthermore, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which further suppresses queue buildup and contributes to the decline in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑106 illustrates the impact of the block size on the average numbers of customers in the block queue for high-priority, low- priority, and overall customers. As increases, the average number of customers in the block queue rises gradually across all priority levels. The increase is most noticeable for and , while the remains relatively low and stable. This indicates that although larger blocks permit more customers per batch, the average block occupancy tends to saturate when the customer arrival rate is equal to the effective service rate of the customer queue. In addition, is smaller than . This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue and the high-priority customers have faster consensus rate than low-priority customers in the block queue, and therefore more low-priority customers remain waiting in the customer queue before being batched. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which contributes to a reduction in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑107 illustrates the impact of the block size on the blocking probabilities for high-priority, low- priority, and overall customers. As increases, the blocking probability decreases across all priority levels. The decline is more pronounced for , which is initially much higher and drops significantly with increasing . This is because larger blocks allow more customers to be served per block generation cycle, thereby reducing the chance of the customer queue reaching its capacity limit, especially for low-priority customers who are more likely to be blocked under limited queue capacity. In addition, is smaller than . This is because the capacity limit of the customer queue for the high-priority customers is no smaller than that for low-priority customers. Additionally, with impatience is smaller than that without impatience. This is because the impatience mechanism causes customers to leave the queue once their impatience threshold is reached, which contributes to a reduction in . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑108 illustrates the impact of the block size on the system throughputs for high-priority, low-priority, and overall customers. As increases, the system throughput increases across all priority levels and then gradually saturates. Both and increase with , with the growth being more significant for low-priority customers. This is because larger blocks enable more customers to be processed per consensus cycle. However, the throughput eventually approaches a limit determined by the customer arrival rate and the impatient rate, which is less than the system processing capacity. In addition, is smaller than . This is because the customer arrival rate of the high-priority customers is smaller than that of low-priority customers. As a result, with impatience is smaller than that without impatience. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑109 illustrates the impact of the block size on the impatient probabilities for high-priority, low- priority, and overall customers. As increases, the impatient probability across all priority levels. This is because larger blocks allow more customers to be served in each service cycle, which reduces the waiting time in the customer queue. As a result, the probability that customers reach their impatience threshold and leave the system becomes lower. In addition, is higher than , as low-priority customers are more likely to experience longer waits due to the non-preemptive priority discipline. As described above,  with impatience is larger than without impatience. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑103: Effect of block size on average waiting time in the customer queue

Figure 5‑104: Effect of block size on average waiting time in the block queue

Figure 5‑105: Effect of block size on average waiting time in the system

Figure 5‑106: Effect of block size on average number of customers in block queue

Figure 5‑107: Effect of block size on blocking probability

Figure 5‑108: Effect of block size on system throughput

Figure 5‑109: Effect of block size on the impatient probability