# Numerical Results

In this chapter, we present the numerical results derived from both the analytical model and the simulation model, covering four distinct blockchain queuing scenarios. Each scenario has two queues, which are customer queue and the block queue, operating under varying combinations of customer class structures and impatience behavior.

The first scenario models a single-class customer system without impatience, serving as a baseline configuration with First-Come-First-Served (FCFS) discipline and no abandonment. The second scenario introduces two classes of customers with non-preemptive priority, distinguishing high-priority and low-priority customers in both queuing and service procedures. In the third scenario, we revisit the single-class setting while incorporating impatience behavior, where customers may abandon the system after a random impatience threshold. Finally, the fourth scenario combines both priority and impatience, modeling a two-class customer system with distinct abandonment rates and non-preemptive priority rules.

Across all scenarios, the blockchain service is influenced by ON/OFF operational states and employs a partial batch policy during block generation. Performance metrics such as throughput, blocking probability, waiting time, and impatient rate (where applicable) are computed and compared under varying parameter configurations. These results provide a comprehensive view of how priority, impatience, and system reliability jointly impact the overall performance of blockchain-based queuing systems.

## Scenario 1: Single-Class Customer without Impatience

The default values are as provided as below: , , , . The maximum capacity of the system is , and the maximum block size is .

### Block size

Figure 5‑1 to Figure 5‑6 show the relationship between various performance metrics and the block size . Both simulation results and analytical results are shown for comparison.

Figure 5‑1 illustrates the impact of the block size on the average waiting time in the customer queue (). As increases, decreases significantly, particularly at smaller block size. This is because larger blocks allow more customers to be served per service cycle, thereby reducing the time customers spend waiting in the queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑2 illustrates the impact of the block size on the average waiting time in the block queue (). As increases, remains nearly constant. This indicates that the time each block spends in the consensus queue is determined by the consensus rate and system state transition rate, and is independent of block size. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑3 illustrates the impact of the block size on the average waiting time in the system (). As increases, decreases significantly, particularly at smaller block size. This is because larger blocks allow more customers to be served per service cycle, thereby reducing the time customers spend waiting in the queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑4 illustrates the impact of the block size on the average number of customers in the block queue (). As increases, initially grows and then stabilizes around constant value. This indicates that although larger blocks permit more customers per batch, the average block occupancy tends to saturate when the customer arrival rate is equal to the effective service rate of the customer queue, where the effective service rate of the customer queue is dependent on the block generation rate, the system transition rate, and the average block size. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑5 illustrates the impact of the block size on the blocking probability (). As increases, steadily decreases. This is because larger blocks allow more customers to be served per block generation cycle, thereby reducing the chance of the customer queue reaching its capacity limit. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑6 illustrates the impact of the block size on the system throughput (). As increases, rises rapidly at first and then gradually saturates. This is because larger blocks enable more customers to be processed per consensus cycle, but the throughput eventually approaches a limit determined by the customer arrival rate, which is less than the system processing capacity. The system processing capacity is decided by the maximum block size, the block generation rate, consensus rate, and system state transition rate. Lastly, the analytical results are in good agreement with the simulation results.

Figure ‑: Effect of block size on average waiting time in the customer queue

Figure ‑: Effect of block size on average waiting time in the block queue

Figure ‑: Effect of block size on average waiting time in the system

Figure ‑: Effect of block size on average number of customers in block queue

Figure ‑: Effect of block size on blocking probability

Figure ‑: Effect of block size on system throughput

### Arrival rate

Figure 5‑7 to Figure 5‑12 show the relationship between various performance metrics and the arrival rate . Both simulation results and analytical results are shown for comparison.

Figure 5‑7 illustrates the impact of the arrival rate on the average waiting time in the customer queue (). As increases, increases steadily. This is because higher arrival rate leads to more customers entering the system, causing increased congestion and longer queuing delays before customers can be batched into blocks. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑8 illustrates the impact of the arrival rate on the average waiting time in the block queue (). As increases, remains nearly constant. This indicates that the time each block spends in the consensus queue is determined by the consensus rate and system transition rate, and is independent of the arrival rate. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑9 illustrates the impact of the arrival rate on the average waiting time in the system (). As increases, increases steadily. This is because higher arrival rate leads to more customers entering the system, causing increased congestion and longer queuing delays before customers depart from the system. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑10 illustrates the impact of the arrival rate on the average number of customers in the block queue (). As increases, increases steadily. This is because a higher arrival rate leads to more frequent block formation and more customers being accumulated in each block, increasing the average number of customers waiting for consensus. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑11 illustrates the impact of the arrival rate on the blocking probability (). As increases, rises sharply, specially beyond . This is because a higher arrival rate leads to more customers being accumulated in the customer queue, increasing the chance that incoming customers are blocked due to limited queue capacity. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑12 illustrates the impact of the arrival rate on the system throughput (). As increases, rises rapidly at first and then gradually saturates. This is because higher arrival rates supply more customers into the system, but throughput eventually becomes limited by the system service capacity, which is decided by the maximum block size, the block generation rate, the consensus rate, and system transition rate. Lastly, the analytical results are in good agreement with the simulation results.

Figure ‑: Effect of arrival rate on average waiting time in the customer queue

Figure ‑: Effect of arrival rate on average waiting time in the block queue

Figure ‑: Effect of arrival rate on average waiting time in the system

Figure ‑: Effect of arrival rate on average number of customers in block queue

Figure ‑: Effect of arrival rate on blocking probability

Figure ‑: Effect of arrival rate on throughput

### Block generation rate

Figure 5‑13 to Figure 5‑18 show the relationship between various performance metrics and the block generation rate . Both simulation results and analytical results are shown for comparison.

Figure 5‑13 illustrates the impact of the block generation rate on the average waiting time in the customer queue (). As increases, decreases significantly. This is because higher block generation rates allow customers to be grouped and processed more frequently, which reduces congestion in the customer queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑14 illustrates the impact of the block generation rate on the average waiting time in the block queue (). As increases, remains nearly constant. This indicates that the time each block spends in the consensus queue is determined by the consensus rate and system transition rate, and is independent of the block generation rate. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑15 illustrates the impact of the block generation rate on the average waiting time in the system (). As increases, decreases steadily. This is because higher block generation rates allow customers to be grouped and processed more frequently, which reduces congestion in the customer queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑16 illustrates the impact of the block generation rate on the average number of customers in the block queue (). As increases, initially grows and then stabilizes around constant value. This indicates that although more frequent block generation can expedite customers service, the average block occupancy tends to saturate because of the arrival rate and the effective service rate reached a state of equilibrium. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑17 illustrates the impact of the block generation rate on the blocking probability (). As increases, decreases rapidly. This is because higher block generation rates allow customers to be served more frequently, reducing the possibility that the customer queue reaches its capacity and causes incoming arrivals to be blocked. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑18 illustrates the impact of the block generation rate on the system throughput (). As increases, rises rapidly at first and then gradually saturates. This is because higher block generation rates enable more customers to be processed per consensus cycle, but the throughput eventually approaches a limit determined by the customer arrival rate, which is less than the system processing capacity. Lastly, the analytical results are in good agreement with the simulation results.

Figure ‑: Effect of block generation rate on average waiting time in the customer queue

Figure ‑: Effect of block generation rate on average waiting time in the block queue

Figure ‑: Effect of block generation rate on average waiting time in the system

Figure ‑: Effect of block generation rate on average number of customers in block queue

Figure ‑: Effect of block generation rate on blocking probability

Figure ‑: Effect of block generation rate on system throughput

### Consensus rate

Figure 5‑19 to Figure 5‑24 show the relationship between various performance metrics and the consensus rate . Both simulation results and analytical results are shown for comparison.

Figure 5‑19 illustrates the impact of the consensus rate on the average waiting time in the customer queue (). As increases, decrease steadily. This is because a higher consensus rate shortens the time block spend in the consensus queue, enabling faster turnover and more frequent admission of waiting customers from the customer queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑20 illustrates the impact of the consensus rate on the average waiting time in the block queue (). As increases, decrease significantly. This is because a faster consensus rate allows blocks to be processed more quickly, thereby reducing the time that blocks spend waiting in the queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑21 illustrates the impact of the consensus rate on the average waiting time in the system (). As increases, decrease significantly. This is because faster consensus processing reduces delays in the block queue, which in turn accelerates the overall service flow and shortens the total time customers spend in the system. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑22 illustrates the impact of the consensus rate on the average number of customers in the block queue (). As increases, decrease significantly. This is because a higher consensus rate enables faster processing of blocks, which reduces queue accumulation and lowers the . Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑23 illustrates the impact of the consensus rate on the blocking probability (). As increases, decrease rapidly. This is because faster consensus rate enables faster processing of blocks, and lowers the probability of reaching the queue capacity that triggers blocking. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑24 illustrates the impact of the consensus rate on the system throughput (). As increases, increase rapidly at first and then gradually saturates. This is because higher consensus rate allows blocks to be processed more efficiently, but the throughput eventually approaches a limit determined by the customer arrival rate, which is less than the system processing capacity. Lastly, the analytical results are in good agreement with the simulation results.

Figure ‑: Effect of consensus rate on average waiting time in the customer queue

Figure ‑: Effect of consensus rate on average waiting time in the block queue

Figure ‑: Effect of consensus rate on average waiting time in the system

Figure ‑: Effect of consensus rate on average number of customers in block queue

Figure ‑: Effect of consensus rate on blocking probability

Figure ‑: Effect of consensus rate on system throughput

### Transition rate (from ON to OFF)

Figure 5‑25 to Figure 5‑30 show the relationship between various performance metrics and the transition rate from ON to OFF . Both simulation results and analytical results are shown for comparison.

Figure 5‑25 illustrates the impact of the transition rate on the average waiting time in the customer queue (). As increases, increases steadily. This is because more frequent transitions from ON to OFF reduce the availability of block generation service, causing longer queueing delays for arriving customers. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑26 illustrates the impact of the transition rate on the average waiting time in the block queue (). As increases, increase steadily. This is because more frequent service interruptions delay consensus processing, resulting in longer waiting times for customer(s) in block queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑27 illustrates the impact of the transition rate on the average waiting time in the system (). As increases, increase steadily. This is because more frequent service interruptions caused by transitions to the OFF state reduce overall availability of block generation and consensus service, leading to longer queueing delays for customers in the system. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑28 illustrates the impact of the transition rate on the average number of customers in the block queue (). As increases, increases steadily. This is because more frequent service interruptions of block generation processing cause more customers to wait in the customer queue while forming batches from the customer queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑29 illustrates the impact of the transition rate on the blocking probability (). As increases, increases steadily. This is because more frequent service interruptions reduce the system’s capacity to process customers, which increases the probability that the customer queue reaches its capacity and causes incoming arrivals to be blocked. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑30 illustrates the impact of the transition rate on the system throughput (). As increases, decreases gradually. This is because more frequent service interruptions reduce the chance for block generation and consensus processing, thereby limiting the rate at which customers are served and ultimately lowering the overall system throughput. Lastly, the analytical results are in good agreement with the simulation results.

Figure ‑: Effect of transition rate on average waiting time in the customer queue in the system

Figure ‑: Effect of transition rate on average waiting time in the block queue

Figure ‑: Effect of transition rate on average waiting time in the system

Figure ‑: Effect of transition rate on average number of customers in block queue

Figure ‑: Effect of transition rate on blocking probability

Figure ‑: Effect of transition rate on system throughput

## Scenario 2: Two-Class Customer without Impatience

The default values are as provided as below: , , , , , , . The maximum capacity of the system is , and the maximum block size is .

### Block size

Figure 5‑31 to Figure 5‑36 show the relationship between various performance metrics and the block size . Both simulation results and analytical results are shown for comparison.

Figure 5‑31 illustrates the impact of the block size on the average waiting time in the customer queue () for high-priority, low-priority, and overall customers. As increases, the overall decreases. The reduction is more pronounced for low-priority customers, while the waiting time for high-priority customers remain consistently low. This is because larger blocks allow more customers to be served per service cycle, thereby reducing the time, especially for low-priority customers who tend to experience longer delays when is small. In addition, the waiting time in the customer queue for high-priority customers is much smaller than that for low-priority customers. This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑32 illustrates the impact of the block size on the average waiting time in the block queue () for high-priority, low-priority, and overall customers. As increases, remains nearly constant for all priority levels. This indicates that the time each block spends in the consensus queue is determined by the consensus rate and system state transition rate, and is independent of block size. Second, the waiting time in the block queue for high-priority customers is smaller than that for low-priority customers. This is because the high-priority customers have faster consensus rate than low-priority customers in the block queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑33 illustrates the impact of the block size on the average waiting time in the system () for high-priority, low-priority, and overall customers. As increases, the overall decreases. The decline is especially significant for low-priority customers, while the waiting time for high-priority customers remains relatively constant. This is because larger blocks allow more customers to be served per service cycle, which benefits low-priority customers who are otherwise delayed by the non-preemptive priority mechanism. In addition, the waiting time in the system for high-priority customers is much smaller than that for low-priority customers. This is because the high-priority customers have non-preemptive priority over low-priority customers in the customer queue and the high-priority customers has faster consensus rate than low-priority customers in the block queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑34 illustrates the impact of the block size on the average number of customers in the block queue () for high-priority, low- priority, and overall customers. As increases, rises gradually across all priority levels. The increase is most noticeable for overall and low-priority customers, while the number of high-priority customers remains relatively low and stable. This indicates that although larger blocks permit more customers per batch, the average block occupancy tends to saturate when the customer arrival rate is equal to the effective service rate of the customer queue. In addition, the average number of block queue for high-priority customers is smaller than that for low-priority customers. This is because the high-priority customers have faster consensus rate than low-priority customers in the block queue, cause more low-priority customers remain waiting in the customer queue before being batched. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑35 illustrates the impact of the block size on the blocking probability () for high-priority, low- priority, and overall customers. As increases, decreases across all priority levels. The decline is more pronounced for low-priority customers, whose blocking probability is initially much higher and drops significantly with increasing . This is because larger blocks allow more customers to be served per block generation cycle, thereby reducing the chance of the customer queue reaching its capacity limit, especially for low-priority customers who are more likely to be blocked under limited queue capacity. In addition, the blocking probability for high-priority customers is smaller than that for low-priority customers. This is because the capacity limit of the customer queue for the high-priority customers is no smaller than that for low-priority customers. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑36 illustrates the impact of the block size on the system throughput () for high-priority, low-priority, and overall customers. As increases, increases across all priority levels and then gradually saturates. Both and increase with , with the growth being more significant for low-priority customers. This is because larger blocks enable more customers to be processed per consensus cycle. However, the throughput eventually approaches a limit determined by the customer arrival rate, which is less than the system processing capacity. In addition, the system throughput for high-priority customers is smaller than that for low-priority customers. This is because the customer arrival rate of the high-priority customers is smaller than that of low-priority customers. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑31: Effect of block size on average waiting time in the customer queue

Figure 5‑32: Effect of block size on average waiting time in the block queue

Figure 5‑33: Effect of block size on average waiting time in the system

Figure 5‑34: Effect of block size on average number of customers in block queue

Figure 5‑35: Effect of block size on blocking probability

Figure 5‑36: Effect of block size on system throughput