# Numerical Results

In this chapter, we present the numerical results derived from both the analytical model and the simulation model, covering four distinct blockchain queuing scenarios. Each scenario has two queues, which are customer queue and the block queue, operating under varying combinations of customer class structures and impatience behavior.

The first scenario models a single-class customer system without impatience, serving as a baseline configuration with First-Come-First-Served (FCFS) discipline and no abandonment. The second scenario introduces two classes of customers with non-preemptive priority, distinguishing high-priority and low-priority customers in both queuing and service procedures. In the third scenario, we revisit the single-class setting while incorporating impatience behavior, where customers may abandon the system after a random impatience threshold. Finally, the fourth scenario combines both priority and impatience, modeling a two-class customer system with distinct abandonment rates and non-preemptive priority rules.

Across all scenarios, the blockchain service is influenced by ON/OFF operational states and employs a partial batch policy during block generation. Performance metrics such as throughput, blocking probability, waiting time, and impatient rate (where applicable) are computed and compared under varying parameter configurations. These results provide a comprehensive view of how priority, impatience, and system reliability jointly impact the overall performance of blockchain-based queuing systems.

## Scenario 1: Single-Class Customer without Impatience

The default values are as provided as below: , , , . The maximum capacity of the system is , and the maximum block size is .

### Block size

Figure 5‑1 to Figure 5‑6 show the relationship between various performance metrics and the block size . Both simulation results and analytical results are shown for comparison.

Figure 5‑1 illustrates the impact of the block size on the average waiting time in the system (). As increases, decreases significantly, particularly at smaller block size. This is because larger blocks allow more customers to be served per service cycle, thereby reducing the time customers spend waiting in the queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑2 illustrates the impact of the block size on the average waiting time in the customer queue (). As increases, decreases significantly, particularly at smaller block size. This is because larger blocks allow more customers to be served per service cycle, thereby reducing the time customers spend waiting in the queue. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑3 illustrates the impact of the block size on the average waiting time in the block queue (). As increases, remains nearly constant. This indicates that the time each block spends in the consensus phase is determined by the consensus rate and system state transition rate, and independent on block size. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑4 illustrates the impact of the block size on the average number of customers in the block queue (). As increases, initially grows and then stabilizes around constant value. This indicates that although larger blocks permit more customers per batch, the average block occupancy tends to saturate due to early block formation under partial batching. Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑5 illustrates the impact of the block size on the blocking probability (). As increases, steadily decreases. This is because larger blocks allow more customers to be served per block generation cycle, thereby reducing the chance of the customer queue reaching its capacity limit from to Lastly, the analytical results are in good agreement with the simulation results.

Figure 5‑6 illustrates the impact of the block size on the system throughput (). As increases, rises rapidly at first and then gradually saturates. This is because larger blocks enable more customers to be processed per consensus cycle, but the throughput eventually approaches a limit determined by the block generation rate and the system’s processing capacity. Lastly, the analytical results are in good agreement with the simulation results.

Figure ‑: Effect of block size on average waiting time in the system

Figure ‑: Effect of block size on average waiting time in the customer queue

Figure ‑: Effect of block size on average waiting time in the block queue

Figure ‑: Effect of block size average number of customers in block queue

Figure ‑: Effect of block size on average number of customers in block queue

Figure ‑: Effect of block size on throughput